# Homework 1: Toby's Answers

**Problem 1)** The director of a California experiment station used to offer visitors a free acre of California. When they gladly accepted, he gave them a glass vial with soil in it, explaining that soil has such a large specific surface that an entire acre's worth of area fit into the vial. How much loam (20% clay, 40% sand, 40% silt), in grams, does it take to make an acre's worth of soil surface area? Assume that the clay is montmorillonite.

Approach: figure out the area of 1 gram, then calculate how many grams are needed.

1 g loam = 0.4 g sand + 0.4 g silt + 0.2 g clay

To calculate the area of 0.4 g sand, I need a representative grain size. Clearly, using the largest possible size (2 mm) will underestimate surface area, and using the smallest (0.05 mm) will overestimate it. You could use the average (1.025) but that's also probably high. Typically in a case like this one uses a non-arithmetic mean. Here I'll use a geometric mean (no problem if you used a different method, but I recommend this one):

$$SandMean = \sqrt{2.0 \times 0.05} = 0.316mm$$

Similarly,

$$SiltMean = \sqrt{0.05 \times 0.002} = 0.01mm$$

Now we can get the area using Hillel's equation 3.17:

$$SphereArea = \frac{6}{\rho_s}\sum_i \frac{c_i}{d_i} = \frac{6}{2.65\frac{g}{cm^3}} \times \left(\frac{0.4g}{0.316mm} + \frac{0.4g}{0.01mm}\right) = 0.093m^2$$

For the clay, we can use Hillel's calculation of 750 $m^2\ g^{-1}$ to get

$$ClayArea = 750m^2 g^{-1} \times 0.2g = 150m^2$$

Combining, 1g of soil has 150.093 $m^2$, with the contribution of the sand and silt small enough that we actually could have ignored it.

1 acre = 0.405 Ha = $10^4$ $m^2$, so we need

$$\frac{0.405 \times 10^4 m^2}{150.093 m^2 g^{-1}} = 27g$$

Depending on the specific values you used, your answer may vary.

**Problem 2)** You are analyzing a soil sample for particle size distribution… (see the Homework handout). I made a mistake in my initial calculations, so I will work this problem out the way I told you to. But if you ever need to do particle size analysis, don't do that "correction" of dividing by 100! And really, the "correction" I suggested wasn't the best, either.

Approach: Get the sand separates as a fraction of the total soil. For the pipette method, you get a diameter from the sample time, and the fraction from the sample mass. The pipette calculations are based on the 1.32 g of soil put into the cylinder, so you need to re-adjust to make this a fraction of the total soil mass.

Solution: Total mass of soil = 0.07 + 0.12 + 0.21 + 0.33 + 0.28 + 1.32 = 2.33 grams.
For the sieving, remember that the soil starts with particles up to 2 mm diameter. What is retained on the 1.0 mm sieve is particles in the 1.0 – 2.0 mm range. Here is a table of the sand fraction:

| Sieve size, mm | Mass retained, g | Particle size, mm | Cumulative % |
|---|---|---|---|
| -- | -- | 2.000 | 100.0 |
| 1.0 | 0.07 | 1.0 | 97.0 |
| 0.5 | 0.12 | 0.5 | 91.8 |
| 0.2 | 0.21 | 0.2 | 82.8 |
| 0.1 | 0.33 | 0.1 | 68.7 |
| 0.05 | 0.28 | 0.05 | 56.6 |
| Pan | 1.32 | Sand fraction | 43.3% |

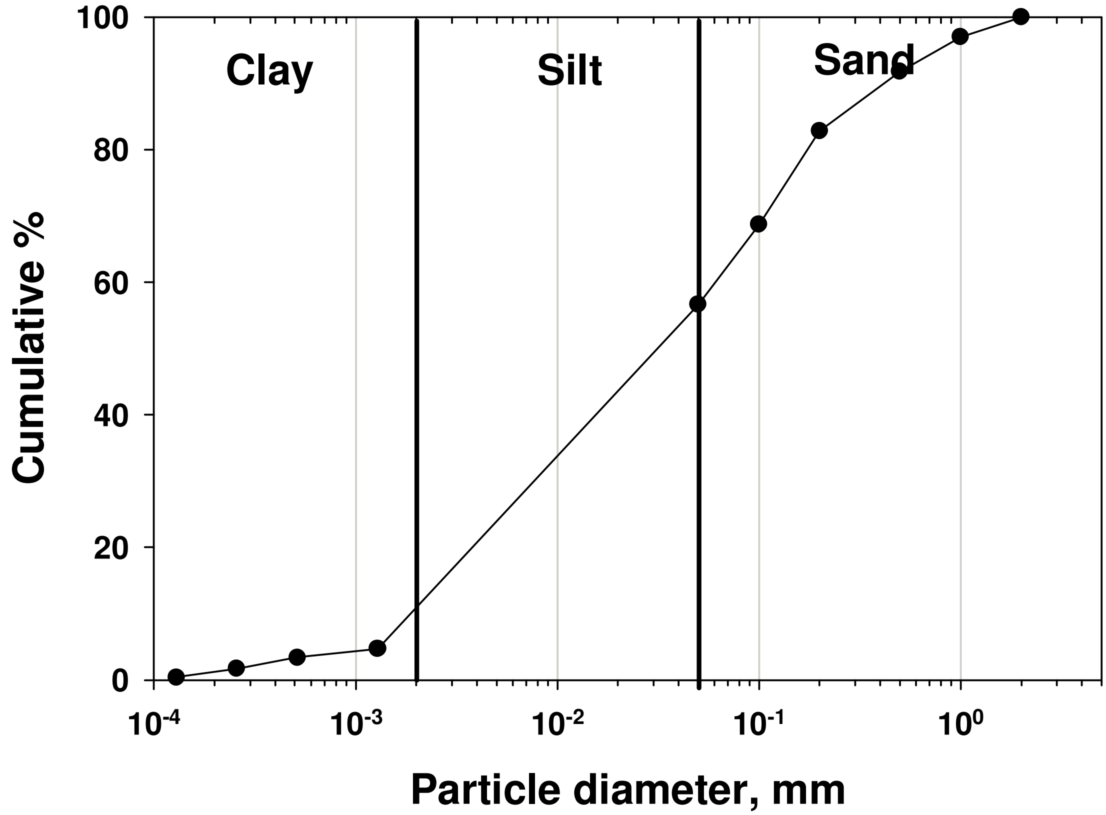
For the pipette, we are working with the 1.32 g that passed all the sieves ("pan"). We assume that the sampling is along a plane (actually closer to a sphere) at depth 20 cm. Using Hillel's formula

$$d = \sqrt{\frac{18d\eta}{(\rho_s - \rho_f)tg}}$$

, we can plug in a few values: d=0.2m, viscosity = 1.25 x $10^{-2}$ kg/m s, g = 9.81 m $s^{-2}$, density difference is 1620 kg $m^{-3}$, and the time is given in the homework handout.
To work out the cumulative percent, it may be easiest to work backwards, from the latest time (finest particles). At that time, 0.0001g of sample is found in 10 ml of fluid; scaling up to the whole liter, 0.01g out of the 1.32 would be suspended, or 0.01 g of the original 2.33g soil sample, for 0.4%. For the previous time, we have 0.04 g still suspended (of which 0.01 is finer) and 0.04/2.33 = 1.7%. So the results are:

| Sample time, s | Sample mass, g | Calculated diameter, mm | *"Corrected" diameter, mm* | Cumulative % |
|---|---|---|---|---|
| 171 | 0.0011 | 0.1286 | *0.001286* | 4.7 |
| 1070 | 0.0008 | 0.0514 | *0.000514* | 3.4 |
| 4280 | 0.0004 | 0.0257 | *0.000257* | 1.7 |
| 17000 | 0.0001 | 0.0129 | *0.000129* | 0.4 |

Plotting this gives the following:




Reading off the lines at the sand/silt and silt/clay boundaries, we have 11% clay, 46% silt, and 43% sand, giving a texture of loam.

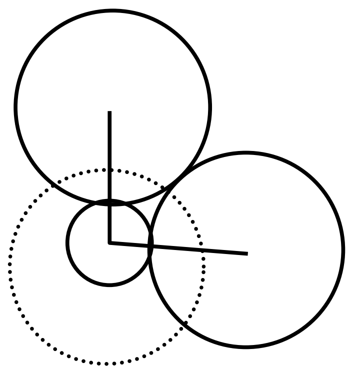
**Problem 3)** Suppose you have packed spheres of radius 1 cm. What is the largest size sphere you can fit inside the matrix if the original spheres are packed cubically? What if they are packed tetrahedrally?

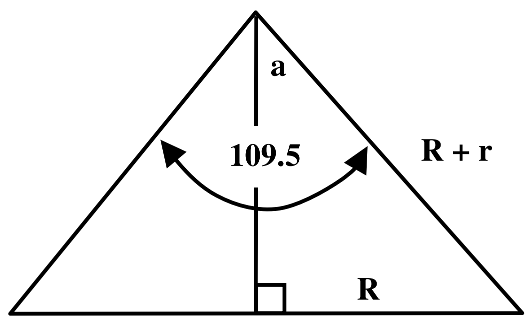
<u>Cubic packing</u>: Consider a "unit cell" of 8 spheres. Each sphere is centered on a corner of the cube. The distance from one corner to the diagonally farthest corner (say, top right front to bottom left back) is 2 cm more than the diameter of the largest sphere that would fit in.

That diagonal across a unit ***square*** is sqrt(2). The diagonal across a unit ***cube*** is sqrt(3) (you can work this out by applying the Pythagorean theorem: using the diagonal of the square, and the unit length of the square, as legs of a right triangle; the cube diagonal is then the hypotenuse).

So the diagonal is $2cm\sqrt{3}$, the diameter of the fitting sphere is 2 cm less than that, or $2cm\sqrt{3} - 2cm$, and the radius is half of that, or $(\sqrt{3} - 1)cm$, or approximately 0.73 cm.

Tetrahedral packing: This is a little trickier. We know that the 4 original spheres in the tetrahedron touch each other, so the distance between any two is 2 cm. We need to find the angle at the center. Looking this up in the CRC handbook or a handy chemistry book, we see that the angle from original sphere center to fitting sphere center to original sphere center is 109.5 degrees. Let's reduce this to a right triangle so we can use trigonometric functions.





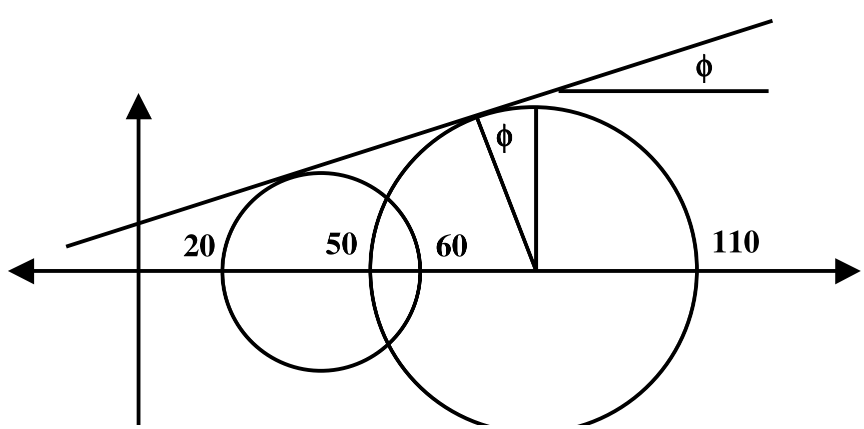

In the figure to the left, we're looking at a cross-section of the tetrahedron, with the fitting sphere inside. We are looking at a cross-section defined by the plane that cuts the three solid spheres through their center. The two other spheres are shown dashed, one precisely behind the other. The angle shown is 109.5 degrees. The right figure now takes this angle, makes it an isosceles triangle, with a sphere center at each apex, and bisects the 109.5 angle to form two right triangles.

Looking at the right triangle to the right, we know that angle "a" must be 109.5/2 = 54.75 degrees, and R = 1 cm. This gives us $\sin(54.75) = \frac{R}{R+r}$, which we solve to obtain r = 0.22 cm.

**Problem 4)** You perform 2 triaxial shear experiments on identical samples of a soil. For the first, the confining pressure is 20 and the load 60 MPa at failure. In the second, the confining pressure is 50 and the load 110 at failure.

a) what is the angle of internal friction?
b) what might be the texture of the soil, and why?

Constructing the Mohr circles, we see this:



The angle of internal friction can be easily **approximated** by finding the slope of the line passing through (40,20) and (80,30). This gives us an angle of

$$\frac{y_2 - y_1}{x_2 - x_1} = slope$$

, and slope = tan(angle), so

$$\phi = \arctan\left(\frac{30-20}{80-40}\right) = 14°$$

But this isn’t actually correct: the line should be tangent to the circles. It shouldn’t pass through the top point on each circle, but rather through a point <angle of internal friction> counter-clockwise from the top point, making it slightly steeper.

The key to an easy solution is noticing that this approximate line and the correct line share the same x-intercept. By definition, the tangent line is at right angles to a radius of the circle, so the radius, the tangent line, and the x-axis form a right triangle. Knowing the distance from the intercept to the center of a circle, we can solve for the angle. The x-intercept is at –40, so using the smaller circle we have a right triangle with corners at (-40,0), (40, 0), and the tangent point. So arcsin(20/80) = 14.47 degrees.

To solve this directly, you could instead use a formula for the slope of the line tangent to two circles. That formula is

$$\left(r^2 - 2rR + R^2 - Y^2 + 2yY - y^2\right) + 2m(y-Y)(x-X) + m^2\left(r^2 - 2rR + R^2 - X^2 + 2xX - x^2\right) = 0$$

where x and X are the x-coordinates of the two circles, y and Y are the y-coordinates, and r and R are the radii. m is the slope, and is the root of the equation – notice the equation has the form of a quadratic in m, so you can solve it with the quadratic equation. Using this formula, we again get an angle of 14.47 degrees.

As for the texture, the X-intercept is negative. The soil has some cohesiveness, so it is a good guess to say that it is not a sandy soil, and has fair clay content. Loam, or anything higher in clay, is a fair guess given what little information we have.