Capillarity

Free water, free water table, cohesion of water.

Interface: energy per unit surface area, derived from unequal forces at an interface.

Interfacial tension on a single drop of water:

Droplet: water pressure pushing out, air pressure pushing in, and interfacial tension along the surface. Suppose we cut a droplet of water in half, somehow compensating for the forces so we really had a half-sphere of water. What is the balance of forces normal to the plane that cuts the drop?

Outward force of water: $F_1 = \pi R^2 P_l$

Inward force of air: $F_2 = \pi R^2 P_a$

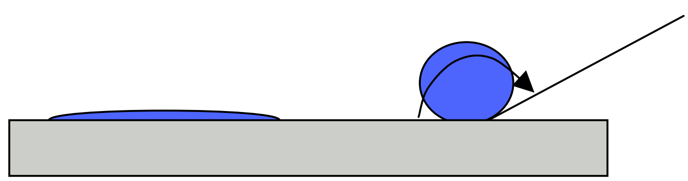
Interfacial force: $F_3 = 2\pi R \gamma$

Because $F_1 = F_2 + F_3$, we can combine them:

$$P_c \equiv P_l - P_a = \frac{2\gamma}{R}$$

We call this $P_c = P_l - P_a$ the capillary pressure P_c (although in this specific case there is no capillary!), and γ is called the interfacial tension.

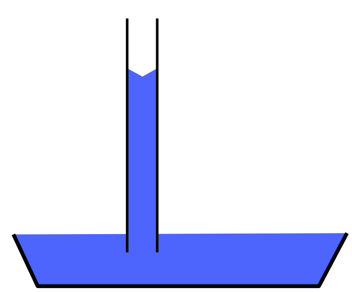
Now we introduce a solid surface, so we have 3 phases: fluid 1, fluid 2, and the solid. Put a drop of fluid 1 (say, water) onto the surface.



These are extreme cases: almost perfect wetting, and almost perfect non-wetting. In general, perfectly clean silica surfaces are perfectly wetting, and many organic surfaces are non-wetting. But of course you don't often find perfectly clean, flat silica surfaces in the soil!

When free water is introduced to a capillary tube of clean glass, the water is observed to rise. Once equilibrium is reached (almost instantly), we can assume forces are at equilibrium and calculate the various forces at work. This derivation follows that of Jury et al., 1991.

We're assuming a spherical shape to the interface, so the water droplet analysis applies here. The radius of the interface is the radius of the tube, so



$$P_c = \frac{2\gamma}{R}$$

The column of water is essentially cylindrical with volume $\pi R^2 H$,
For a capillary rise of height H.
The upward force, due to interfacial tension, is

$$F_{up} = P_c A = 2\pi R\gamma$$

while the downward force is

$$F_{down} = Mg = V(\rho_l - \rho_a)g = \pi R^2(\rho_l - \rho_a)Hg$$

Setting the 2 forces equal, we have

$$H = \frac{2\gamma}{(\rho_l - \rho_a)gR}$$

If the contact angle is not 0, we modify this to give

$$H = \frac{2\gamma\cos(\alpha)}{(\rho_l - \rho_a)gR}$$

What would happen if you stuck a plastic tube in, with a wetting angle of 180 degrees rather than 0? Or if you stuck the glass tube into a bucket of a non-wetting fluid?

How would you determine α and γ for a specific fluid/fluid/solid combination?

How abut non-circular pores? Aren't most pores non-circular?

1) we use "equivalent diameters"
2) Young-Laplace equation: $P_c = \gamma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$
3) In the corners themselves, more rise. Squeezed parallel plate example (this then showed up as the last question in Exam 1).

Advancing versus retreating contact angles – surface roughness.

Structured water at a solid interface.