

Part 1: Building Blocks: (36 pts total)

1: In the following table, each line (row) concerns a single equation. Fill in the missing entries (2 pts, each). If the equation doesn't have a name, state what it describes (e.g., Coulomb's envelope).

Name	Equation	Physical situation described
Richards equation	$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \left[K(\theta) \frac{\partial \psi}{\partial x} \right]$	<i>Saturated and unsaturated, transient and steady-state flow of water through soil</i>
<i>Capillary equation (or capillary rise eqn.)</i>	$h = \frac{2\gamma \cos(\alpha)}{(\rho_w - \rho_a)gR}$	<i>Height of rise of water in a capillary tube</i>
<i>Components of total potential</i>	$\psi_T = \psi_g + \psi_m + \psi_p + \psi_o + \dots$	Total potential of water in soil
Advection-dispersion equation	$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2} - v \frac{\partial c}{\partial x}$	<i>Diffusive-dispersive transport of a solute in a soil under 1-D, steady flow.</i>
<i>Darcy's law</i>	$q = K \frac{\Delta h}{\Delta x}$	Steady-state, saturated flow through soil
<i>Definition of (specific, or differential) water capacity</i>	$C(\theta) = \frac{\partial \theta}{\partial \psi}$	<i>The specific water capacity is the the slope of the water retention curve.</i>
Equation of continuity	$\frac{\partial \theta}{\partial t} = - \frac{\partial q}{\partial x}$	<i>In – Out = change in storage: mass is conserved in transient flow.</i>

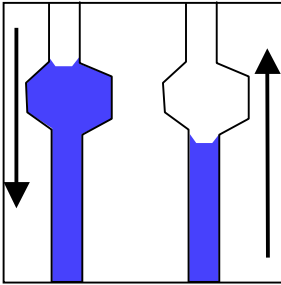
2: Give units for the following (1 pt. each). Use base SI units where you can.

Concentration (e.g., of a solute): <i>Mass/volume, such as kg/l</i>	Dielectric constant: <i>unitless</i>
θ <i>vol / vol = unitless</i>	Hydraulic conductivity: <i>m/s</i>
Dispersion coefficient: <i>m² / s</i>	Diffusion coefficient: <i>m² / s</i>
Flux density: <i>m³ / m² s = m/s</i>	Flux: <i>m³ / s</i>

Part 2: Comprehension: (5 pts each; 30 pts total)

3) Explain the inkbottle effect.

The “inkbottle effect” is one mechanism theorized to cause hysteresis between the drainage and wetting curves of the soil water characteristic. Consider a pore that has a narrow throat at the top and bottom, but is wider in the middle. If it is full of water and draining, water will be retained in the narrow neck by capillary forces, but eventually the meniscus radius will become unstable and collapse, draining the pore. So during drainage, water is held by the top radius. During wetting, the larger radius of the center acts as a capillary barrier, delaying wetting – so wetting is controlled by the larger radius.



4) Describe one situation in the lab, and one in the field, where you would encounter pneumatic potential.

In the lab: pneumatic potential (in the form of compressed air) is used to push water out of soil samples. This is how most water retention curves are made.

In the field, high- and low-pressure weather systems change the atmospheric pressure on the soil water, slightly changing the total water potential. This is generally a minor effect, but measurable.

5) How does a neutron probe measure water content?

A neutron probe emits fast neutrons, and detects slow neutrons. Hydrogen atoms are effective at slowing neutrons, so the number of slow neutrons detected is a good indicator of the amount of water in the soil.

6) What is “soil water potential”?

Soil water potential is a measure of the energy state of the water in the soil. It is defined as the energy needed to move the water from a reference state (pure, free water) to the location and state in question, reversibly and isothermally.

7) Why is the scaling of a dispersion model important?

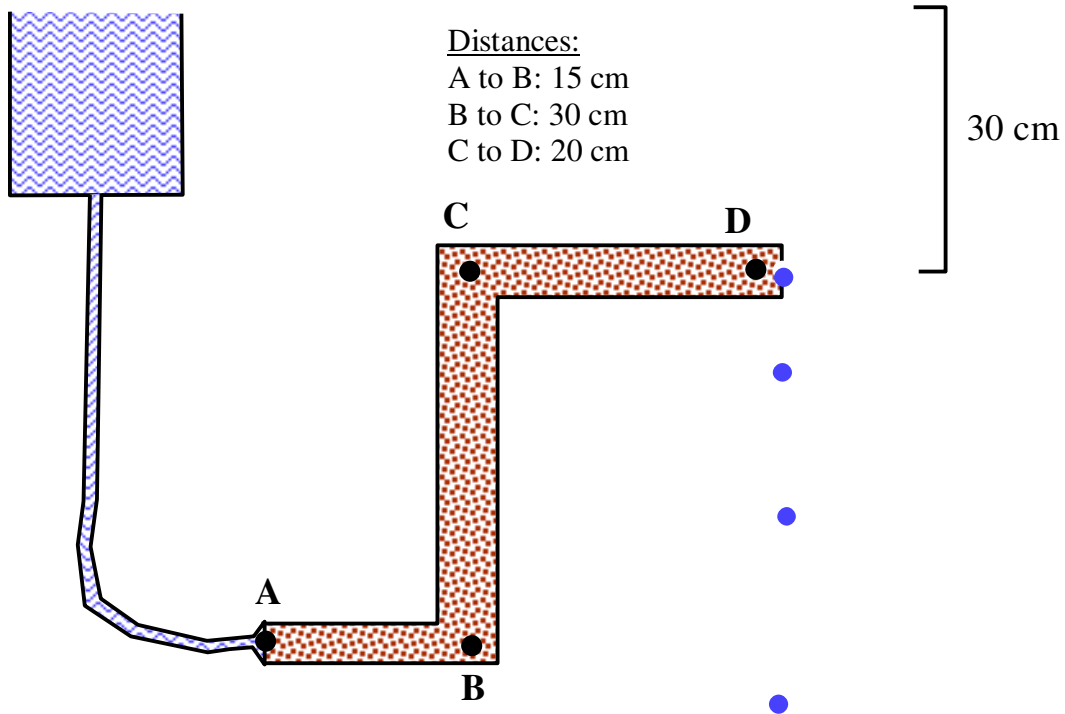
The scaling behavior of a dispersion model tells you whether the standard deviation of the solute plume will move linearly with time, or with the square root of time. This is important because 1) it tells you something about the mechanisms at work, and 2) if your scaling is wrong, then any prediction you make at a different scale (in time or space) may be very wrong.

8) Why is short-time cumulative infiltration proportional to square root of time, while long-term infiltration is proportional to time?

Short-time infiltration is largely driven by the matric potential, while long-time infiltration is driven by gravity. So short-time infiltration is like horizontal infiltration, which nicely conforms to a diffusion model (“soil water diffusivity”) with the wetting front advancing with square root of time. Longer-time infiltration is linear with time because the hydraulic gradient approaches unity. These behaviors are seen in the Philip’s infiltration equation $i = S \sqrt{t} + Kt$.

Part 3: Application: (15 pts each; 45 pts. total)

9) Water is moving out of a 25 cm tall reservoir through a large tube, into a soil column, then out through a hole at the end.



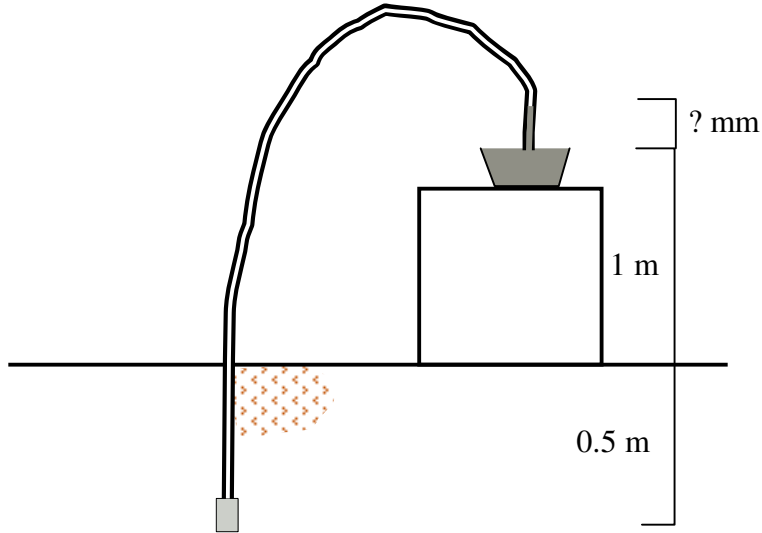
Fill in the table. Use A as the gravitational reference.

Location	Ψ_T	Ψ_g	Ψ_m
A	60	0	60
B	53.1	0 cm	53.1
C	39.2	30	9.2
D	30	30	0

What is the ratio of the flux density when the water level is at the top of the reservoir, to the flux density when the water level is at the bottom of the reservoir? Assume $K = 0.2$ mm/hr.

The area and length of the column are constant, so only the potential gradient is changing. The drop in head changes from 30 to 5 cm, and since $q \sim K \sim dh$, the flux must drop by 6.

10) You have a mercury tensiometer buried at 0.5 m below the ground surface, as shown in the (oddly familiar) diagram below. You know that the water table is 3 m below the ground surface. If the system is at equilibrium, what is ψ_m at the tensiometer? What is the height of rise of the mercury, currently given as “? mm”? Recall that $\rho_{\text{Hg}} = 13,600 \text{ kg m}^{-3}$.



If the system is at equilibrium, then total potential at the water table = total potential at the tensiometer. The tensiometer is 2.5m higher than the water table (where matric potential = 0), so matric potential at the tensiometer must equal -2.5 .

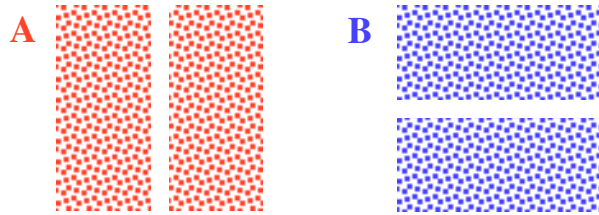
Now set up the tensiometer calculation: what is pulling down on the left must equal what is pulling down on the right.

$$2.5 \text{ m (matric)} + 1.5 \text{ m (water)} + x \text{ (ht of mercury)} = 13.6 x \text{ (ht of mercury)}$$

$$4.0 \text{ m} = 14.6 x$$

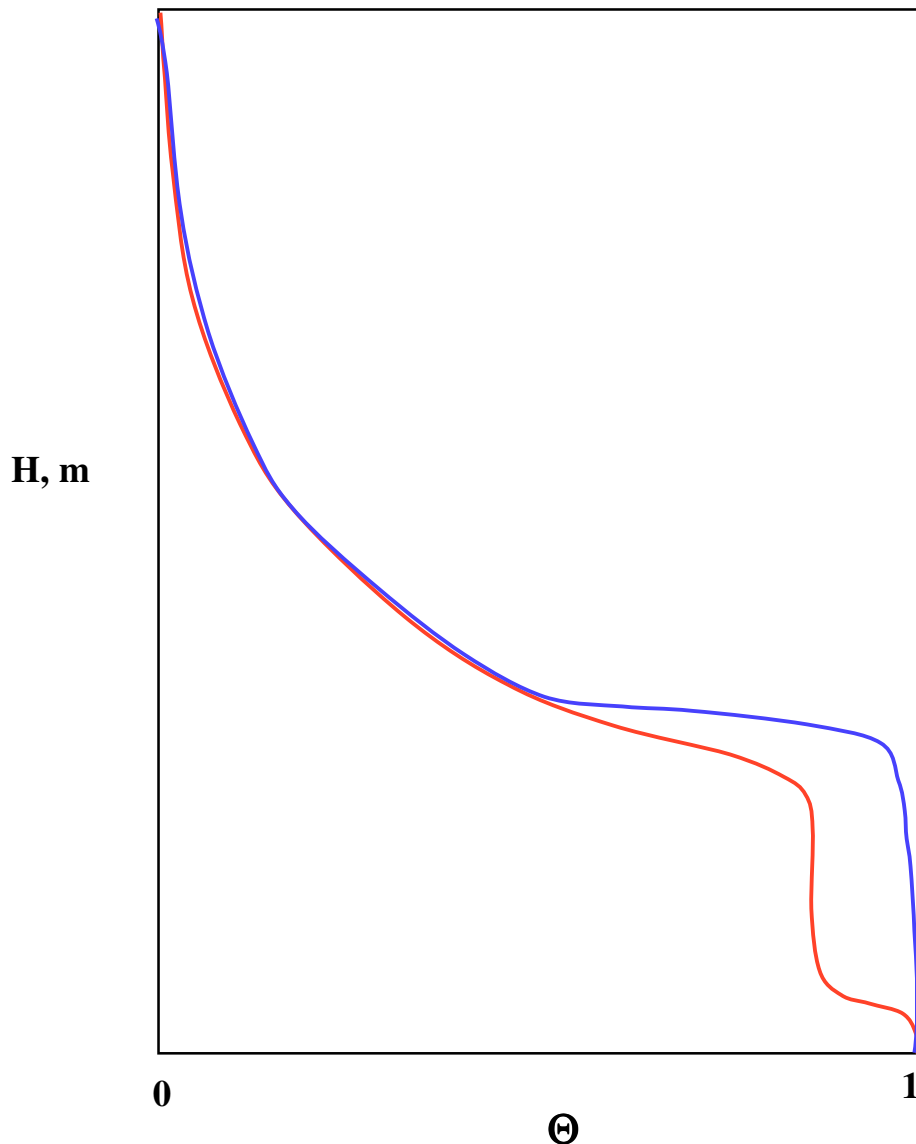
$$x = 4/14.6 = .274 \text{ m or } 27.4 \text{ cm mercury.}$$

11) We saw in the homework how the continuity of large pores affects the saturated hydraulic conductivity. Does it also affect water retention? Consider the following two two-dimensional soils:



Suppose that the matrix porosity is 0.5, pores in the matrix are 1.0mm in radius, the “macropore” is 1.0 cm in radius, and the sample is 5 cm tall. On this page, sketch the primary drainage curve for the two soils and label them **A** and **B**; on the next page, do the same for the primary wetting curve. *Don't worry about exact values* for the potential axis: I'm more interested in the shapes of the curves.

Primary Drainage Curve:



Primary Wetting Curve:

