

Part 1: Building Blocks: (32 points total)

1: In the following table, each line (row) concerns a single equation. Fill in the missing entries (2 points each). If the equation doesn't have a name, state what it describes (e.g., Coulomb's envelope).

Name	Equation	Physical situation described
Richards equation	$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x} \left(K(\theta) \frac{\partial \psi}{\partial x} \right)$	<i>Transient and steady-state, saturated and unsaturated, horizontal and vertical flow of water in soil.</i>
Capillarity equation	$h = \frac{2\gamma \cos \alpha}{(\Delta\rho)gr}$	<i>Height of rise of water in a capillary tube.</i>
<i>Definition of thermal diffusivity</i>	$D_T = \kappa / C_v$	<i>Steady-state and transient flow of heat.</i>
<i>Stokes' law</i>	$v = \frac{d^2 (\Delta\rho)g}{18\eta}$	<i>Settling of a solid sphere in a viscous fluid, driven by the difference in density between the solid and the fluid.</i>
<i>Philip's infiltration equation</i>	$I = s\sqrt{t} + Kt$	<i>Cumulative infiltration with time, where infiltration is limited by the soil rather than by precipitation.</i>
<i>Definition of damping depth</i>	$d = \sqrt{2D_T / \omega}$	<i>Transient heat flow into the soil, particularly heat flow driven by a sinusoidal thermal surface input.</i>
Root zone water balance equation	$\Delta S + \Delta V = P + I + U - (ET + R + D)$	<i>In – Out = Change in storage, with more detailed enumeration of the components of each class.</i>
<i>Surface energy balance equation</i>	$J_n = LE + A + S + M$	<i>Partitioning of total radiant energy at the soil surface into components: latent heat, sensible heat, soil heating, and miscellaneous.</i>

2: Give units for the following (1 pt. each). Use base SI units where you can.

Volumetric heat capacity: $J m^{-3} K^{-1}$, or $Cal m^{-3} K^{-1}$	Damping depth: cm or m
κ $J / m K s$	Intrinsic permeability: cm or m
Dispersion coefficient: m^2 / s	Diffusion coefficient: m^2 / s
Viscosity: $kg m^{-1} s^{-1}$	Osmotic potential: <i>Depends: N / m^2, m, or J / kg (any one of these was fine)</i>
Pressure: $N m^{-2}$	Density: $kg m^{-3}$
J_n : $W m^{-2}$	θ $m^3 m^{-3}$, or <i>unitless</i>
Sorptivity: $cm s^{-0.5}$ (you can derive it from Philip's equation on page 1)	Albedo: <i>unitless</i>

Part 2: Comprehension: (5 pts each; 45 pts total)

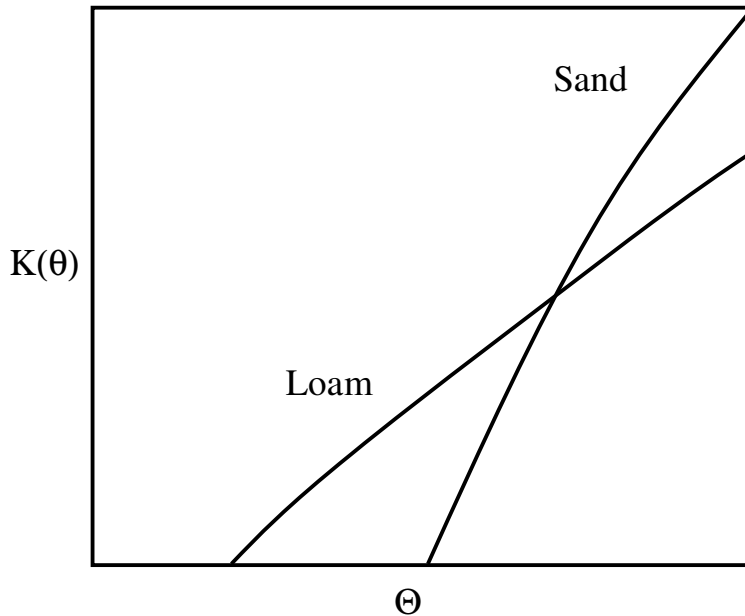
3: In Homework #3 there was a question about a fire at the soil surface and heat moving downward. I solved the problem using an equation from Crank's *The Mathematics of Diffusion*. Why is it OK to use a diffusion equation for a heat flow problem? Be specific.

Heat flow is mathematically similar to diffusion, even though the physical processes are different. The basic equation that describes them is of the same form:

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2}$$

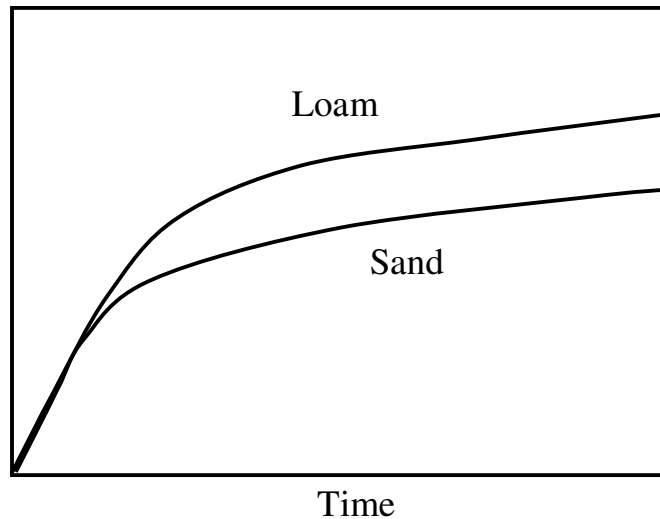
and so solutions for one apply to the other as well.

4: In the box below, sketch the unsaturated hydraulic conductivity function $K(\theta)$ for a sand and a loam soil. Assume that the Y-axis is logarithmic, and that both soils have the same porosity.



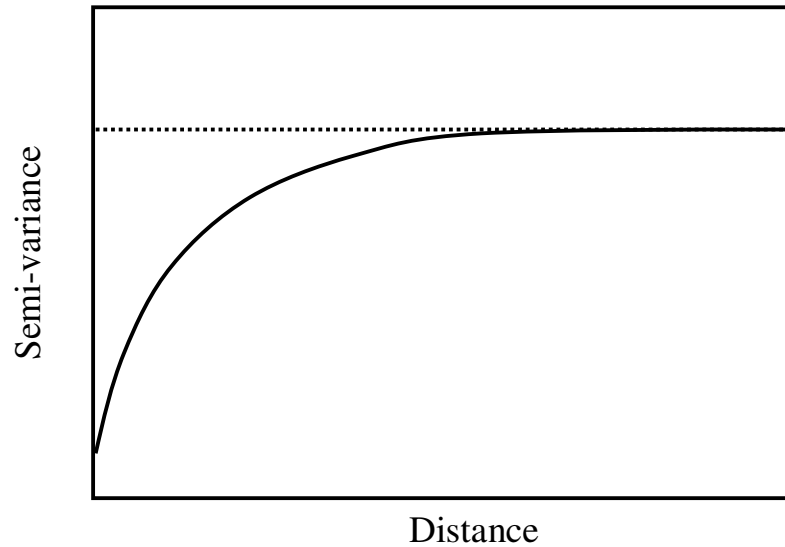
The $K(\theta)$ curve is almost straight on a log-linear plot, but the sand starts at a higher K and drops faster.

5: Sketch cumulative evaporation from a sandy and a loamy soil over a 2-week period.



The two start at the same rate (atmosphere limited), but soon the sand dries at the surface and evaporation becomes diffusion-limited (the reason is related to the graph in the previous question). Notice that the question asks for a cumulative curve.

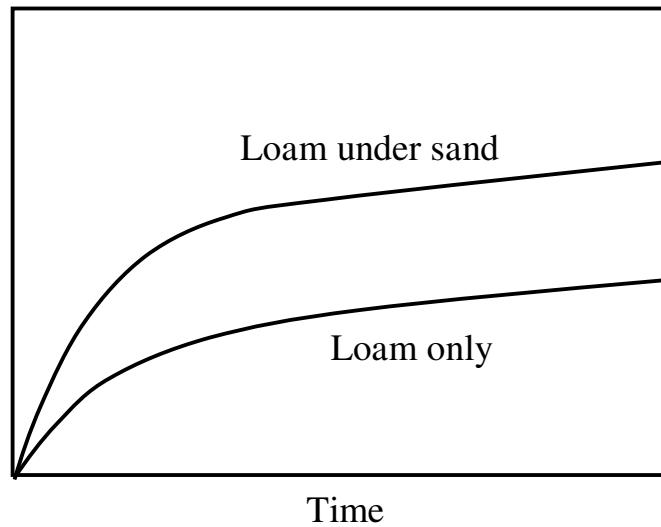
6: Draw a semi-variogram, label the axes, and explain the meaning of the graph.



The meaning of the graph is simply that things that are close together are more similar than things that are far apart (or, more precisely, that the properties of objects that are close together are more closely related than those of objects that are farther apart).

I did not ask for labels of the nugget, sill, or range, though I was pleased that many students were able to provide them.

7: Draw curves to represent 1) cumulative infiltration into a uniform loamy soil, and 2) cumulative infiltration into a similar soil that has a 10 cm sand layer above the loam. Assume the soils are at the same antecedent wetness, and that no crusting occurs.



The two will eventually arrive at the same infiltration rate (parallel lines), but the upper sand layer will allow faster initial infiltration. Again, this is a cumulative graph.

8: Why does dispersion of solutes in soil increase approximately linearly with water velocity?

It has been observed that, in the field, solute dispersion often scales linearly with velocity. An explanation can be found by analogy with Taylor's experiments with solutes in a tube. Recall that Taylor's dispersion resulted from adequate mixing between the fast and slow streamlines in the tube. If the flow is fast enough, the mixing will be inadequate, and so dispersion will be greater. Translation to the field: if the velocity is greater, then there is less mixing and so more dispersion.

Notice that this question is about velocity, NOT about time. The question of whether dispersion scales with time or square root of time is entirely different.

9: Hillel speaks of the soil surface zone as acting like a preferential valve that allows infiltration to be much more rapid than evaporation. Explain how this "valve" functions.

Infiltration is limited by the capacity of the soil to take in water, which means essentially that it is limited by the hydraulic conductivity. K is a strong case of water content. When water is infiltrating, it makes a saturated wetting front, and so in effect the water raises its own conductivity.

When water is evaporating, the evaporation rate may be limited by the ability of the soil to transmit water to the surface. Because evaporation dries the surface, the hydraulic conductivity decreases as evaporation progresses.

Conclusion: the $K(\theta)$ function is how the valve works: high surface K for water coming in, and low surface K for water leaving.

10: Why is there an optimal wetness for compaction? Describe a practical application of this fact.

Compaction is about pushing air out of the soil.

If the soil is too wet, there is less air to push out, and a compacting force would face the harder task of pushing out water, which has a much higher viscosity.

If the soil is too dry, none of the pressure is borne by the fluid – it's all carried by the particle-particle contacts. Without lubrication, compaction is less efficient.

If the extremes of wet or dry are hard to compact, an intermediate wetness will be optimal. The actual value of this optimal wetness, for a given compacting force, can be evaluated with a Proctor test.

Practical application: if you are compacting a soil for a foundation, you must make sure it isn't too wet, or too dry, before you start. You'll often see soil being watered during highway construction for this very reason.

11: What are the principle mechanisms of, and barriers to, soil aeration?

Aeration is about oxygen moving into the soil and, to a lesser degree, carbon dioxide moving out.

The main mechanisms are diffusion and advection. Diffusion is generally considered the more important mechanisms, and is much faster in the air phase than in the water phase. Soil aeration via advection is either thermally driven, as in diurnal soil heating and cooling which changes the density of the air ("soil breathing", which affects only a very limited depth), or meteorologically driven, as in changing air pressure due to wind or weather fronts.

The barriers to soil aeration are essentially all about continuous air pathways: such pathways increase aeration, and their absence decreases it. So high soil wetness, high soil bulk density, lack of soil structure, and a lack of large pores (e.g., because the soil is fine-textured) will all decrease soil aeration.

Part 3: Application: (10 pts each; 50 pts. total)

For all the calculations, assume that the following values apply:

Water viscosity: $1.25 \times 10^{-2} \text{ kg m}^{-1} \text{ s}^{-1}$ Interfacial tension: $7.3 \times 10^{-2} \text{ kg s}^{-2}$
 Particle density: 2650 kg m^{-3} Water density: 1000 kg m^{-3}
 Gravitational acceleration = 9.81 m s^{-2} Wetting angle = 0°

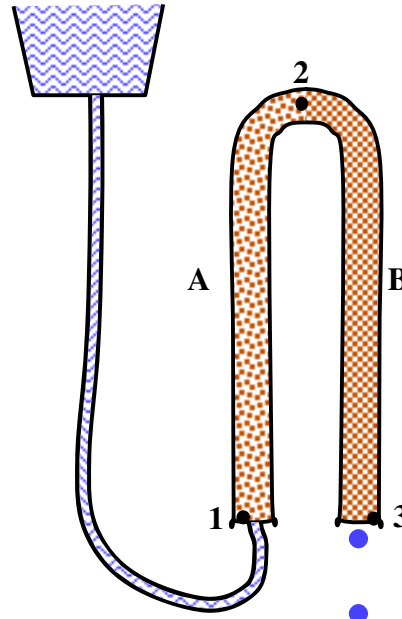
12: You have the physical setup sketched below. The water level is maintained constant at the top of the reservoir. Soil A has a saturated hydraulic conductivity 4 times greater than that of soil B. Fill in the table.

Each column is 40 cm long.

Distance from water table to bottom of column is 60 cm.

First fill in the gravitational column, which is purely based upon height.

The matric potential at 3 is 0, because water is dripping out. The matric potential at 1 is 60, because we assume equilibrium between the reservoir and the column (essentially, we are assuming infinite hydraulic conductivity of the tube).



Knowing that total = gravitational + matric, fill in the totals for locations 1 and 3. Now the only unknowns are at location 2.

*We have a potential drop of 60 cm across the column, and by Darcy's Law we know that $q = K * \text{total potential gradient}$. By continuity we know that $q_A = q_B$. We also know that $K_A = 4 * K_B$. Combining this knowledge, $4 * \text{gradient A} = \text{gradient B}$.*

So the total gradient drop across A AND B is 60 cm, of which 1/5 is across A and 4/5 is across B. In other words, the part of the column with the lower conductivity needs more potential in order to get the same total flux.

Location	Total Potential =	Gravitational Potential +	Matric Potential
1	60	0	60
2	48	40	8
3	0	0	0

13: Suppose you have the following data from measuring particle sizes:

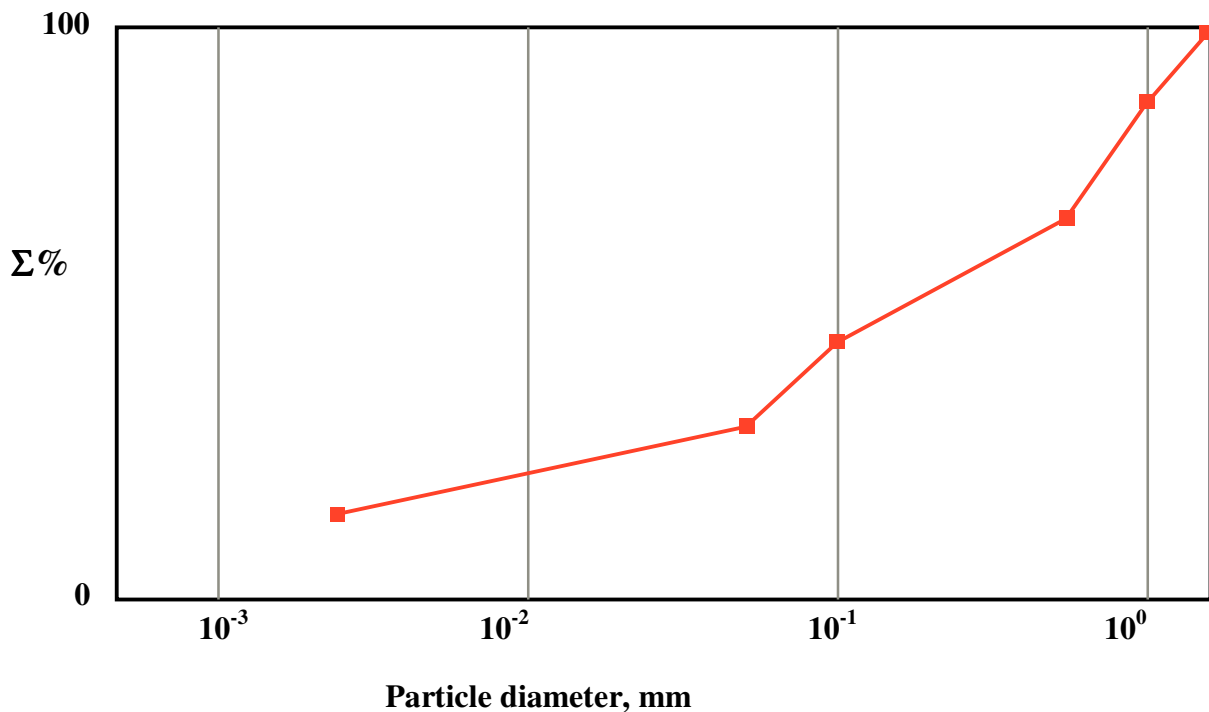
Particle diameter range, mm	Mass, g	Cumulative percent
1.0- 2.0	3.2	100
0.5 - 1.0	3.7	85
0.1- 0.5	4.8	68
0.05 - 0.1	4.2	45
0.002 - 0.05	2.6	25

Assuming that the soil is 13% clay, fill in the rest of the table, and plot the data below.

The Mass column sums to 18.5 g, but there is another 13% in clay. So the original sample was 21.26 g soil.

The cumulative plot goes from 100% at 2 mm (particles > 2 mm diameter are stones, not soil) to 0% at some smaller size. So we can enter 100 at 2.0 mm. Fill in the rest of the table by subtracting the mass of the next fraction from 21.26, then dividing by 21.26 g.

For each (range, percent) pair, we plot the point at the top of the range. Notice that the X-axis is logarithmic, and (for example) the 0.5 point is not halfway between 0.1 and 1.0!



14: Briefly describe 4 different situations we have encountered in soil physics which can reasonably be described by the diffusion equation.

- *Diffusion of a solute in water*
- *Diffusion of air through soil*
- *Movement of heat through soil*
- *Horizontal infiltration, as given by the Bruce & Klute experimental setup*
- *“Early time” vertical infiltration, as given by Philip’s infiltration equation*
- *Dispersion of a solute as described by the ADE*
- *Diffusive exchange of a solute between mobile and immobile regions, as given in the MIM*
- *Diffusion of solute between the walls and center during flow of a fluid in a tube (Sir Geoffrey Taylor’s experiment)*

15: Briefly describe 3 different situations encountered in soil physics to which the capillary rise equation is applicable, if only by analogy.

- *Height of rise of water in a capillary tube (of course!)*
- *Height above the water table of the capillary fringe*
- *The water retention characteristic (by analogy)*
- *Relationship between the water retention curve and the pore size distribution (by analogy)*
- *Concepts of unsaturated flow (by analogy)*

16: Briefly describe at least 3 different situations encountered in soil physics in which continuity and/or connectivity are critical.

- *Soil structure: K_{sat} is determined by a combination of pore size distribution, and pore continuity and connectivity. Remember the interstate, and flow through the cubes with 2 pores each.*
- *Aeration: the rate at which oxygen can diffuse deep into the soil depends strongly on the presence of continuous air-filled pathways.*
- *The water retention curve: a major difference between the curve and its translation into a pore size distribution is that large pores can't drain (replace water by air) unless there is an air-filled pathway to them.*
- *Drainage of the landscape: the continuity of high K (= high wetness) pathways in the field allows faster drainage.*