Soil heat flow (2)

Some people confuse heat capacity and thermal conductivity, so here is a brief clarification. Heat capacity is simply the capacity of a substance to store heat: how much energy does it take to raise the temperature of a unit amount of the substance one degree. Water, in particular, has a high heat capacity. This is why water has a moderating effect on climate: the heat stored during the warm season in lakes and the ocean (for example) is released during the winter.

From the viewpoint of soil physics, adding water to a dry soil roughly triples its volumetric heat capacity. A nice thing about heat capacity is that the components are simply additive: $C = f_a c_a + f_m c_m + f_o c_o + f_w c_w$. The overall volumetric heat capacity C is a linear function of water content.

Thermal conductivity is more dynamic, and so (like any transport process) it depends both on the component parts, and on their **arrangement**. De Vries' classic *Physics of plant environment* (1975) develops a thermal conductivity concept taken from electrical theories. This theory, still in common use today, explains some of why thermal conductivity is not a linear function of wetness, but (because it is an electrical analog) it is not a full explanation.

Here is a quick explanation of how the arrangement of the component parts (water, air, and solid particles) is important to thermal conductivity. A plot of κ versus θ shows a rapid increase in thermal conductivity with just a small addition of water, then a more gradual increase. When the soil is completely dry, heat moving by conduction (the primary mechanism in the soil) moves from one grain to another through point contacts. When a small amount of water is present, it collects at these contact points (asperities, pendular structures), making the thermal contact larger than a single point. This greatly increases the thermal conductivity. Once these contact points are broadened, further additions of water simply decrease the volume of air (a good insulator) and reduce the distance between pathways. This is a smaller effect, and so the increase of κ with θ is not as steep.

Diffusivity is conductivity divided by capacity, so D_T is the nonlinear k divided by the linear C. As it happens, D_T increases steeply with a small amount of water, then remains approximately constant. This is handy because, over small changes in water content, we can assume a constant D_T without incurring too great an error. D_T, being a composite property, is the most convenient for calculations and predictive modeling, so this is a nice simplification.

We have now seen diffusion equations in 3 contexts: diffusion of a solute, hydraulic diffusivity, and thermal diffusivity. Here are some of the similarities and differences:

Similarities:

1) Each case has a primary, linear transport equation, a conservation equation, then a combined, more general secondary equation.
2) Each case has a conductivity, a capacity, and a diffusivity, where the diffusivity is defined as conductivity divided by capacity.
3) In each case, what is being moved is distinct from the driving force itself.

Case	Molecular diffusion	Movement of heat	Movement of water
What is moved	solute	heat	water
Driving force	Concentration gradient	Thermal gradient	Total potential gradient
Primary equation (1-dimensional form)	Fick's first law $J_d = -D_0 \, dc/dx$ (Hillel's eqn. 9.7)	Fourier's law $q_h = -\kappa \, dT/dx$ (Hillel's eqn. 12.8)	Darcy's law $q = -K \, dH/dx$ (Hillel's eqn. 7.7)
Conservation or continuity equation (1-dimensional form)	$\frac{\partial c}{\partial t} = -\frac{\partial J_d}{\partial x}$ (Hillel's eqn. 9.11)	$\rho c_m \frac{\partial T}{\partial t} = -\frac{\partial q_h}{\partial x}$ (Hillel's eqn. 12.9)	$\frac{\partial \theta}{\partial t} = -\frac{\partial q}{\partial x}$ (Hillel's eqn. 7.18)
Secondary equation	Fick's second law $\frac{\partial c}{\partial t} = D_s \frac{\partial^2 c}{\partial x^2}$ (Hillel's eqn. 9.13)	$\frac{\partial T}{\partial t} = D_T \frac{\partial^2 T}{\partial x^2}$ (Hillel's eqn. 12.15)	Richards' equation $\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left[D(\theta) \frac{\partial \theta}{\partial x} \right]$ (Hillel's eqn. 8.22)
Definition of "diffusion" coefficient	$D_s = D_0 \theta / \tau$ (Hillel's eqn. 9.8)	$D_T = \kappa / C$ (Hillel's eqn. 12.13)	$D(\theta) = \frac{K(\theta)}{\partial \theta / \partial \psi}$ (Hillel's eqn. 8.19)

Differences:
1. Hydraulic conductivity is a strong non-linear function of wetness, while the other conductivities are much weaker functions of their respective intensities.

2. Only for molecular diffusion are the conductivity and diffusivity coefficients identical. The theta and tau modify the diffusion coefficient to work in a porous medium, but there is no diffusion capacity term as such.

3. The Richards' equation is written here in horizontal (no gravity) form – without this limit, it wouldn't look nearly as much like a diffusion equation.

4. The water capacity term is more complex than the heat capacity term.