Homework 3

Due Friday, April 25. ________________Toby________________
Show all work. Name
You may work with others if you like, but please list their names if you do.

Toby's answers in italics.

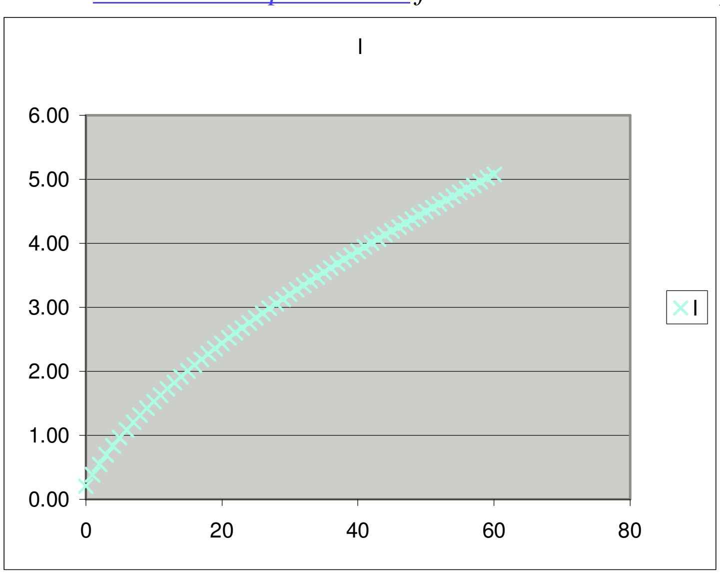
1) If a soil has $\theta = f_a$, which is greater: the permeability of the air phase, or the permeability of the water phase? Explain your reasoning.

Air and water will compete for the pores, and air will end up in the larger pores. Other things being equal, the larger pores have greater flow (via Poiseuille's law), so you would expect the air phase to have greater permeability. This difference between air-filled and water-filled porosity will be greater when the pore size distribution is wider, and/or the soil is more structured.

Opposing opinion: if the total porosity is low, then the air phase may be discontinuous. But the water phase (the wetting phase) is always continuous. In this case, continuity wins over pore size, and the water phase would have greater permeability.

2) Suppose that infiltration proceeds in accordance with Green and Ampt's (1911) model. Let porosity = 0.45, ponding depth = 0 cm, Ksat = 1.3 cm/hr, antecedent volume wetness = 0.22, and suction at the wetting front = 43 cm.
Plot cumulative infiltration over a 1 hour period, starting when the wetting front is 5 cm below the soil surface. Show your work (I recommend using a spreadsheet). What was the minimum rainfall rate needed to sustain this infiltration rate? What was the cumulative depth of water infiltrated?

See the homework 3 spreadsheet for details. Here is the plot:

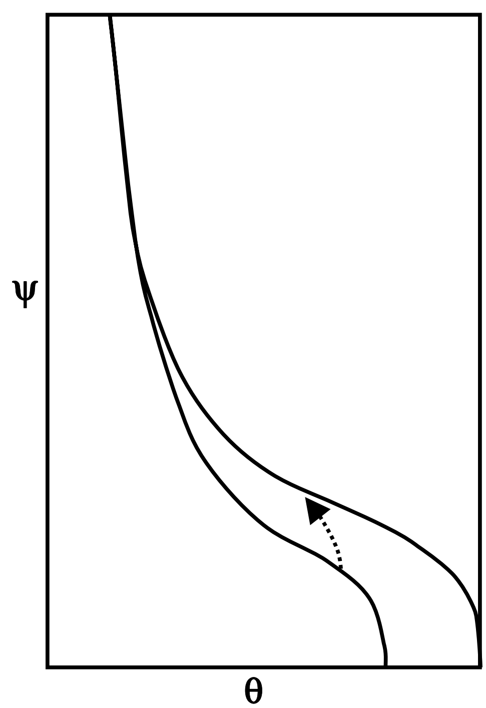


The X-axis is in minutes, and the Y-axis is in cm of water infiltrated. The initial infiltration rate is approximately 12.6 cm/hr, so any less than that would result in less total infiltration (infiltration being precipitation-limited rather than soil-limited). Approximately 5.1 cm of water infiltrated.

For those who like more of a challenge, what value do you get for Philip's sorptivity, using the "data" from the Green and Ampt prediction?

This question was based upon the supposition that your spreadsheet would be able to do multiple regression – even Lotus-123 could do that 15 years ago! But MS Excel seems to lack this ability. So I can only use the Philip equation for short time (with the sqrt(time) term only, no linear time term). Using simple regression, with sqrt(time) as the independent variable and cumulative infiltration as the dependent variable, I get an r^2 of 0.96, and sorptivity of 0.6.

3) Explain the influence of hysteresis on evaporation.



Suppose evaporation starts after infiltration ends. Infiltration is a wetting process, so the soil is on the wetting curve. Now evaporation starts, so the soil must shift to the drying curve. This is done on a so-called scanning looped (dashed line with arrow).

The scanning loop has a large change in potential for a small change in wetness – it is steeper than the drying curve itself. So in the early stages of evaporation, the atmosphere has to "do more work" to extract a small amount of water than it would have had to if soil were not hysteretic.

I appreciate Mario's comment that the capillary-driven overnight rewetting from below can contribute to this effect: it moves the soil from a drying curve during the day to a wetting curve at night, and means that for a given wetness the soil is at a lower potential.

4) Given Wien's law, what temperature should an object be in order to emit microwaves? Given your result, how can a microwave oven heat food?

According to Hillel (page 610), the microwave range begins at 1 mm. Wien's law says that $\lambda_{max} = 2900/T$, for λ in microns and T in Kelvins. Plugging in numbers, I get 2.9 K, or approximately the mean temperature of the universe (the cosmic microwave background has a mean of about 3 K). In other words, this is the peak wavelength emitted by a very cold object.

So how does a microwave oven heat food, if it is mimicking an object near absolute zero? The wavelength (in one sense) doesn't matter: if the oven gives off more radiation than it absorbed, then objects inside it will have a net energy gain regardless of the wavelength. And, as it turns out, microwaves are efficiently absorbed by water, a chief component of most food.

5) A fire is started on the surface of a soil having thermal diffusivity $D_h = 5.5 \times 10^{-3}$ cm^2/s. How long will it take the heat pulse to reach an earthworm 40 cm below the soil surface?

This can be readily solved using the sine-wave equation. The question is about time, so we are looking for the phase shift between a peak at the surface and a peak at 40 cm depth.

From the sine-wave solution, the phase shift is z/d, for z=depth (here, 40 cm), and d=damping depth. If we think of a normal diurnal wave moving downward, then $d = sqrt(2D/\omega)$. Plugging in numbers, $\omega = 7.27 \times 10^{-5}$ /s (see Hillel) for a diurnal cycle, and $D=5.5 \times 10^{-3}$ cm^2/s (given above). Solving, d=12.3 cm. Now z/d = 40/12.3 = 3.25, slightly more than π (which is half a period, or 12 hours). So the answer is slightly greater than 12 hours. Letting $\omega t = z/d$, t = 44731 s = 12.4 hours.

But I don't like this answer, because the damping depth it uses assumes a diurnal (daily) heating and cooling cycle, and there is nothing in the question to justify that. And by extension, why should the heat care how long its pulse is? Shouldn't it move at the same speed regardless?!

So I worked out a different solution, based on Crank's 1-D solution for an instantaneous source (see the "fire" page in the Homework 3 spreadsheet). Crank says that, for an instantaneous source, the concentration (or temp increase, in our case) as a function of depth and time is

$$C(x,t) = \frac{M}{2\sqrt{\pi D t}} e^{-x^2/4Dt}$$

So if we suppose that the fire is an instantaneous source, and then at some later time it is put out and the soil surface temperature is instantaneously returned to "normal", then the heat pulse will be represented by the difference between two of these equations, offset in time by the duration of the fire. Using this approach, I get a pulse travel time of approximately 7.4 hours. Those who are interested may want to ponder why the diurnal cycle's peak appears to travel more slowly.

6) For the same soil, at what depth in the soil profile would you find the lowest temperatures around July 10?

Here we're looking at an annual cycle – July 10th is approximately the time of the hottest day of the year (in the Northern hemisphere). Notice that the annual damping depth should be sqrt(365), or about 19, times greater than the daily depth. So if problem 5 was approximately half a cycle (π), then the damping depth here should be about 19 times the damping depth in problem 5, or about 233 cm. Plugging in numbers, I get a calculated value of 235 cm.

*So: half a cycle difference is π, so we have $\pi = z/d$. Again, putting in numbers we have $z = \pi d$ = 738 cm. Notice the factor of 19 again: 40 (from problem 5) * 19 is approximately 738 cm.*

Quasi-practical application: if you want to use a geo-buffered heat pump to both heat and cool your house, figure out the thermal diffusivity of your soil, and bury the heat exchange coils at the depth you just calculated: – precisely out of phase with the seasons. Then you'll be using the coldest portion of the soil to cool your house in the summer, and warmest to heat your house in the winter.