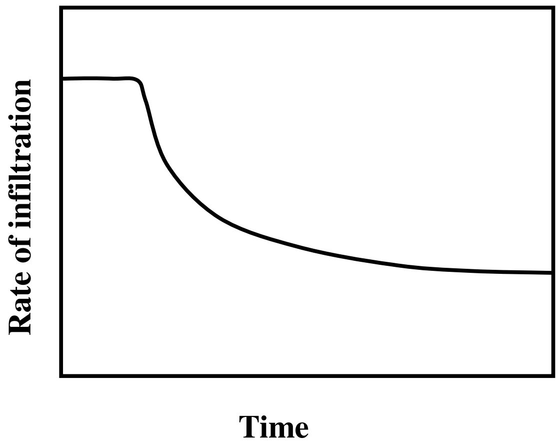
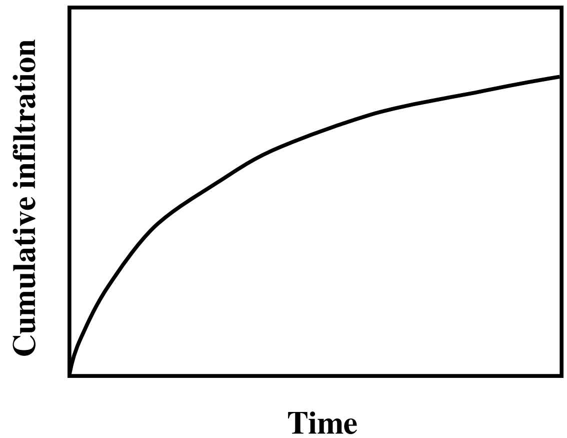
Infiltration

Infiltration is the movement of water downward into the soil through the soil surface. This is a necessary step for soil water recharge. But infiltration is limited by both surface properties, and by the ability of the soil matrix to absorb new water.

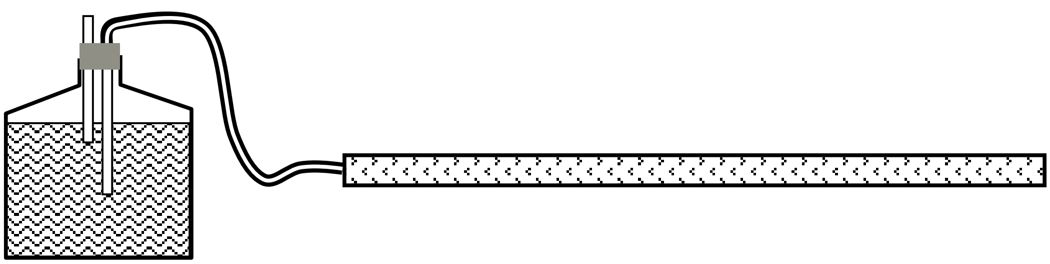
Empirically, we observe that infiltration usually proceeds like this:





We will first look at soil infiltrability to see what intrinsic soil properties contribute to this shape, then we'll look at surface properties.

The simplest case is horizontal infiltration under constant boundary conditions. This kind of experiment was popularized by Kunze and Kirkham, and by Bruce and Klute: typically you'll see it called a Bruce and Klute setup because they wrote the chapter on it in the original *Methods of Soil Analysis*. The basic setup is this:



where the bottle is a Mariotte bottle that delivers a small positive pressure to the mouth of the soil tube. Once flow is established, the bottle (or air inlet tube) is lowered so as to give zero pressure.

[Aside: how does a Mariotte bottle work? Notice that, for water to leave the bottle, it must be replaced by air. Pulling out water will therefore also involve pulling air down the air inlet tube, below the water surface. So the head is actually determined by the bottom of the air inlet tube, not by the water level itself. Some nice pages on this are http://www.uswcl.ars.ag.gov/exper/mariotte.htm and http://lawr.ucdavis.edu/classes/ssc100/mariotte_bottle1.pdf]

The wetting front moves slowly through the soil tube, and can be measured in a variety of ways. For example, the progress of the wetting front can be measured by simple observation if the soil tube is transparent. The soil wetness can be measured using a gamma ray device. The most usual method is to make the soil tube out of many smaller tube segments that are stuck together, and then as some specific time, before the wetting front has reached the far end of the tube, the tube is separated into its many segments. Each segment is then weighed, dried, and weighed again to determine the soil wetness.

The analysis is best explained in terms of the concept of soil water diffusivity, which (as you will recall) is the product of the differential (or specific) water capacity and the unsaturated hydraulic conductivity. That is,

$$D(\theta) = K(\theta) / \frac{d\theta}{d\psi} = K(\theta)\frac{d\psi}{d\theta}$$

Recall from the unsaturated flow chapter that we can use this concept of soil water diffusivity (NOT diffusion!) to formulate the Richards equation as something like a diffusion equation:

$$\frac{\partial\theta}{\partial t} = \frac{\partial}{\partial x}\left[D(\theta)\frac{\partial\theta}{\partial x}\right],$$

which has only two variables, theta and x. In the unlikely case that D were the same for all values of theta encountered we could simplify it further to

$$\frac{\partial\theta}{\partial t} = D\frac{\partial^2\theta}{\partial x^2}$$

which is the form of Fick's second law, the standard formulation for diffusion. Hence the name "diffusivity" – the resemblance to diffusion is purely mathematical.

Diffusion has a characteristic signature: the process propagates in proportion to the square root of time. If this derivation is worth anything, then in the Bruce and Klute experiment we should see the wetting front advance with the square root of time – and we do! So since we apparently have the math right, we get greedy and ask, is there anything else we can figure out about the soil's properties from this kind of experiment?

Ludwig Boltzmann introduced a mathematical trick that is useful here. The so-called Boltzmann transformation declares a new variable lambda, such that

$$\lambda(\theta) = x\sqrt{t}$$

This transformation allows us to rewrite the <u>partial</u> differential equation as an <u>ordinary</u> differential equation (don't ask), giving:

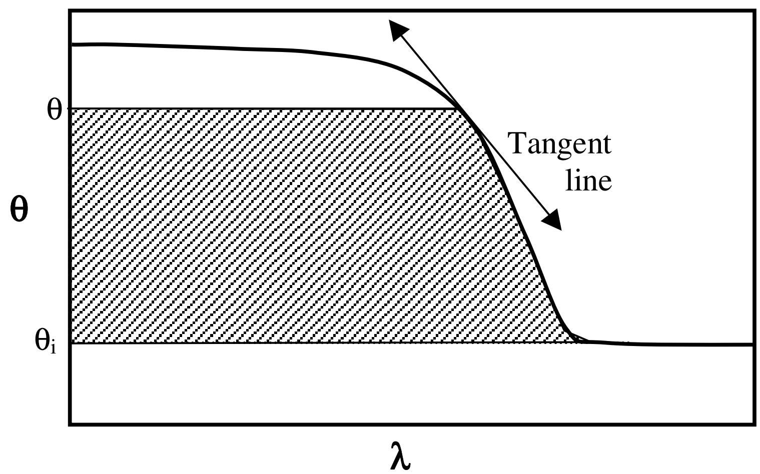
$$-\frac{\lambda}{2}\frac{d\theta}{d\lambda} = \frac{d}{d\lambda}\left[D(\theta)\frac{d\theta}{d\lambda}\right]$$

which again has only 2 variables – theta and lambda – rather than 3 (theta, x, and t) in the original.

Because x (distance to the wetting front) should be proportional to sqrt(t), we can use this lambda to "normalize" the equation: theta(x) at ANY time should, when plotted as theta(lambda), give the same curve. This curve can be used to parameterize the D(theta) function:

$$D(\theta) = -\frac{1}{2}\frac{d\lambda}{d\theta}\int_{\theta_i}^{\theta} \lambda d\theta$$

That is, the value of D(theta), for a given value of theta, is given by the product of the slope of the theta-lambda curve at the desired value of theta, and the area under the curve ("under" with respect to the theta axis) between the desired theta and the initial theta. Schematically:



In summary, by treating the Richards equation like a diffusion equation, we have 1) discovered (and experimentally confirmed) that horizontal infiltration does indeed behave mathematically like a diffusion process, and 2) found a way to get values for the D(theta) function.

So, how do we apply this analysis to vertical infiltration in the real world? Consider that in horizontal infiltration the driving force is the matric potential of the soil, while for vertical infiltration we have a gravity potential contribution as well. Intuitively then, we can say that infiltration has a sqrt(time) component that is due to the matric effects, and a time component that is due to gravitational effects. This leads us to the Philips equation for infiltration,

$$I = s\sqrt{t} + At$$

where s is the so-called "sorptivity" of the soil, and A is related to K_{sat} (Philips says that roughly, A = K/3). For short times, the capillary component of infiltration dominates, while for long times the gravity term is more important. The precise definition of short- and long-term is not simple, and I'll leave it to Philip to explain; for our purposes it suffices to note that matric effects will last longer in soils that are dry and fine-textured, while gravity effects will be important more quickly in soils that are initially wetter and coarser-textured.

If we are restricting ourselves to “short” times, then, the first term (the sqrt(time) term) is the only one that matters. This form is the one given in Hillel. The two-term model is more general and works for longer times as well.