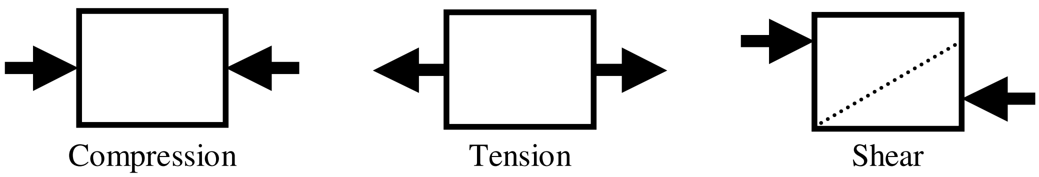
Soil Mechanics: basic concepts and Mohr's circle

Soil mechanics is an applied branch of rheology, deformable body mechanics, and the study of strength of materials. The basic question in these fields is, how does a material behave when subjected to stress?

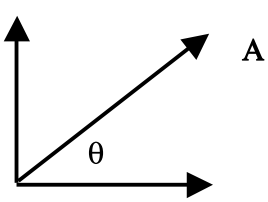
Several "ideal" materials are recognized, for example brittle, elastic, plastic, and viscous. Soil is not ideal, and behaves partially like each of these. This pages focuses on the brittle aspects of soil behavior. A brittle material fails (breaks, shears) under stress.

Imagine two forces acting on a body. If the forces are opposed and act toward the body, the body is under compression. If the forces are opposed and act away from the body, the body is under tension. If the forces are not co-linear, the body experiences some shear, plus either compression or tension:



Soil, like most earth materials, is much stronger with respect to compression than it is to tension and shear. This is not true for all materials: steel, for example, is stronger with respect to tension than to compression.

Now recall that a vector of arbitrary orientation can be "decomposed" into two vectors orthogonal to each other. For example:



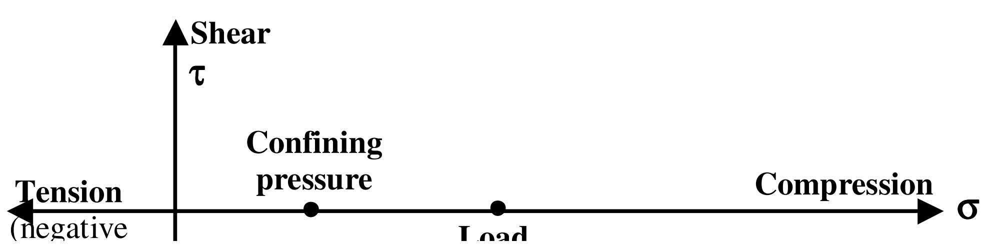
$\boldsymbol{A} = \boldsymbol{A}\ sin(\theta) + \boldsymbol{A}\ cos(\theta)$

where the angle θ is determined by the orientation of the chosen coordinates. Likewise, two orthogonal vectors can be combined into a single vector:

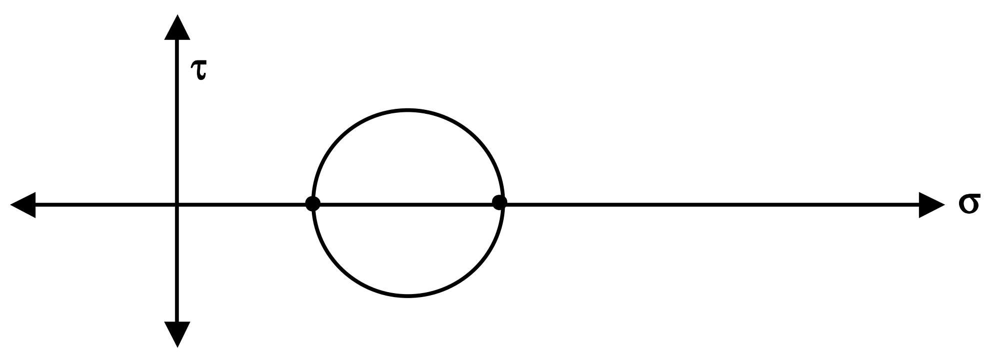
$\boldsymbol{A}\ sin(\theta) + \boldsymbol{A}\ cos(\theta) = \boldsymbol{A}$

where now the angle θ is given by the relative magnitudes of the two vectors.

If we imagine a point in the soil, and consider the forces acting on it, we see two: load (vertical stress), and confining pressure (resistance to squashing from the soil around it). The load is greater than the confining pressure, and the two are orthogonal to each other. We can plot these two points in stress space (tension and compression [symbol: σ] on the X axis, and shear [symbol: τ] on the Y axis) thus:

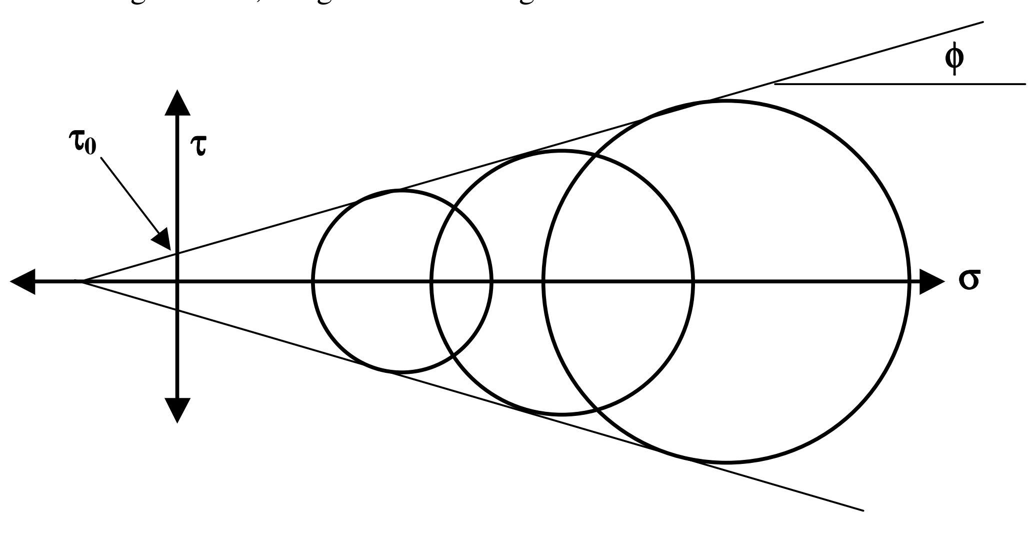


When we look at the forces across a vertical plane through our point, confining forces are acting but load is not. Likewise, load is expressed fully on a horizontal plane, but confining pressure is not. By rotating our frame of reference, the forces change from compression only to compression with shear. That is, exactly how the forces are represented depends on our point of view. But the soil doesn't share out point of view, so it "experiences" any or all of these. So we generalize the stress depiction (load and confining pressure) by showing it at all possible orientations. This results in a circle, which represents the **state of stress at that point**. This idea of showing the state of stress in a body was developed by Mohr, so this diagram is called Mohr's circle.



If we increase the load, the soil will eventually deform - generally (for soil) by shearing. This is called failure. Let's plot the circle at the critical pressure, exactly at failure. Then points inside the circle are possible, and points outside the circle are not.

If we increase the confining pressure, the soil gains strength (resists shearing better). For each initial confining pressure, we can plot another circle. It turns out that, if we plot a bunch of these circles for a given soil, we get the following:



The lines tangent to the circles represent the generalized failure criterion for this material: states of stress inside this "envelope" are possible, and states outside are not. This diagram is called the failure envelope, or the Coulomb envelope.

A state of tension is possible in this specific soil (the diagonal lines have a negative X-intercept: negative compression is tension). This means that the soil has cohesiveness. Generally this means that clay, cementation, or organic matter is present in the soil: a pure sand has no cohesiveness (no resistance to tension), and so it would have a zero or positive X-intercept.

The upper diagonal line can be given as $\tau = \tau_0 + tan\ \phi$, which is a general definition of the failure envelope. This is essentially the equation that Hillel gives on page 348. The angle, ϕ, represents the angle of internal friction. The angle of the shear plane in the sample, θ, is given as $\theta = \pi/4 + \phi/2$.